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Factors Affecting the Control of Unstable Combustors

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Active control systems for the attenuation of pressure oscillations in unstable combustors have been under development by a wide variety of investigators. Whereas many of these applications have been in simplified, bench-scale combustors, some significant efforts have been made in realistic systems including full-scale gas turbine engines. Results from these efforts, although generally positive, have been varied. As the technology has matured, the understanding of the factors that currently limit its capability has improved. Some of these factors and how they may be quantified through experiments, models, or combined approaches are addressed. The application focus was lean, premixed combustors for industrial gas turbines. Both gas- and liquid-fueled systems were considered. In particular, the effects of actuation system time delay and actuated fuel mixing are examined, and the fundamental limits of control in systems with time delay are considered. The results indicate that the mixing of the actuated fuel with the remainder of the premixed reactants is significant in determining control authority. The effects of system time delays are shown to result in a peak splitting phenomenon, which limits the degree to which pressure oscillations can be reduced. A further examination of the fundamental limits of control demonstrates that it is not possible to arbitrarily decrease the level of pressure oscillations using linear controllers. The factors that limit the achievable reduction level are the time delay, the combustor damping, and the actuator bandwidth.

Nomenclature

b	=	on level for solenoid valve
G_c	=	controller transfer function
G_0	=	transfer function representing combustor, fuel line, and valve dynamics
k	=	exponent determining rolloff of open-loop transfer function outside control bandwidth
N	=	random-input describing function
S	=	sensitivity function
$\Delta\omega_1$	=	performance bandwidth
$\Delta\omega_2$	=	control bandwidth
ε	=	required attenuation level for sensitivity function over performance bandwidth
μ	=	mean fuel/air ratio
σ	=	standard deviation of valve command
σ_r	=	real part of unstable pole
τ	=	delay
Φ_{ii}	=	power-spectral density function of the input disturbance
Φ_{pp}	=	power-spectral density function of the combustor pressure
ϕ	=	fuel/air equivalence ratio
ω	=	frequency
ω_b	=	lower boundary control bandwidth
ω_c	=	higher boundary control bandwidth

Introduction

THE lean, premixed combustor designs utilized in low-emissions industrial gas turbines are often prone to combustion instabilities. Significant efforts to suppress these instabilities using active control techniques have been reported. Researchers at United Technologies Research Center[1,2] have demonstrated up to 16-dB suppression of combustion instabilities in a full-scale single combustor and a 6.5-dB attenuation in a three-nozzle sector combustor. Engineers at Siemens kWU[3,4] have deployed an active instability control system on a full-scale engine, resulting in reductions in fluctuating pressure of as much as 17 dB. ABB/Alstom investigators[5] have performed considerable work on a laboratory-scale combustor, yielding suppression levels of up to 12 dB. Other organizations have demonstrated similar levels of success in other premixed combustors, including the U.S. Department of Energy,[6] Honeywell, Inc.,[7] and Westinghouse/Georgia Institute of Technology.[8]

Based on these successes, it is clear that this technology holds promise as a means for attenuating combustion instabilities. However, the factors that affect (and possibly limit) the performance of active instability control systems have not been fully investigated. In particular, it has been observed that the achieved reduction of pressure oscillation varied between experiments from 6 to 20 dB. Moreover, in some cases the attenuation of oscillations at the primary frequency was accompanied by excitation of oscillations at some other frequencies.[1,9–11]

The number of factors that can determine the effectiveness of an active instability control system is large. These factors can be categorized as follows: 1) combustor dynamics, 2) actuation system, 3) sensing, and 4) controller/algorithm. A more detailed discussion of these categories follows along with a discussion of which of them this paper will address.

Combustor Dynamics

The dynamic description of the pressure oscillations to be controlled is important to the effectiveness of the control system. This includes the fundamental issue of whether or not the system is dynamically unstable, that is, in a limit cycle, or whether it is linearly stable and driven by noise. Large, coherent pressure oscillations are possible in either case. Another important factor is the mechanism (or mechanisms) through which the unsteady heat release and pressure couple with each other. For the control system to be effective, it is important to understand and prioritize these mechanisms to devise actuation schemes to interfere with them. If multiple mechanisms are at play, it may be difficult to deal with more than one at once. Some mechanisms may not lend themselves to practical actuation schemes or may be better suited to passive control approaches, such as acoustic resonators.[12] Multiple instability modes

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and the interaction between those modes (especially nonlinear interactions) may also compromise the performance of a control system. Further issues with how the combustor pressure oscillations behave as a function of engine operating conditions and engine transients will also affect controller performance.

Actuation System

For the actuation system to be truly effective, it must interfere with the root-cause physics that lie behind the instability. The most important consideration in actuation system design is developing an understanding of what these physics are. It is often difficult to sort out cause/effect relationships from measurements made on an unstable system. The number of potential coupling mechanisms that could be at work in any one system is very large. Modeling and detailed experimentation are often required to acquire this information. Once the intent of the actuator has been specified, that is, how it will interfere with the causal physics, the practical considerations of the actuator and the actuation system must be addressed. In most systems to date, the fuel flow rate has been controlled via some sort of high-speed valve. This valve must be fast enough to modulate fuel flow at the required frequency and, potentially, over a range of frequencies. This will set specifications for the valve's bandwidth and natural frequency. In addition, it must have a flow capacity that is consistent with the supply pressure and flow rate goals of the system, but it still must represent the metering orifice of the system. The interaction of the actuator with the controller is also an important factor. In valves with hysteresis or nonlinear response, position feedback may be required. Proportional control systems will require proportional valves with a wide dynamic range of flow capacity. Whereas the valve is a crucial component of the actuation system, the remainder of the system cannot be overlooked. This includes the plumbing system and the fuel injection system. The plumbing system acoustics can be important and can even be exploited[6,13] for both liquid and gaseous fuels. The capacitance of the fuel delivery system can also have a severe effect on the time variations in the delivery of fuel to the combustor. This is especially true in gaseous-fuel systems, but can also be a factor when air is trapped in liquid-fuel systems. The fuel injection technique may also affect actuator authority. This includes atomization, mixing and transport effects. These phenomena can act to attenuate and/or delay the fuel pulse delivered by the actuator, causing the response at the flame front to be minimal. In some systems, multiple actuators may be required. The placement of these actuators[3] and their coordination are also significant elements to consider.

Sensing

Whereas the technology for sensing combustion instabilities may be fairly mature (high-response pressure transducers or optical chemiluminescence measurements), the implementation of these sensors in a control system can be critical to the system's performance. Note that, although it is relatively simple to measure pressure or heat release oscillations in an unstable system, it may be much more difficult when a control system is successfully minimizing those oscillations. Once again, a knowledge of the causal physics is important. Sensors must be placed at the proper locations relative to the acoustic mode shape to identify properly phase information and to maximize signal/noise ratio. Multiple sensors may also be required to identify or accommodate changes in mode or mode shape. These sensors must be well matched, in terms of both amplitude and phase. Filtering, signal processing, and averaging may also be necessary depending on the nature of the sensed signal. Secondary sensing of the actuation system or external parameters (fuel flow rate, inlet temperature, etc.) may also be required to set control system parameters.

Controller/Algorithm

Designing a control algorithm that processes a pressure sensor signal(s) and sends a command signal a fuel valve(s) to quench the oscillations would be a routine task, if the combustor operating conditions were fixed, the transport delay small (much less than the acoustic period), the actuator authority and bandwidth adequate, and a model of pressure response to the fuel valve command were available. In such a case, the control algorithm would need to provide an appropriate phase shift of the pressure signal. This could be done using several approaches[2,3,9,11,14–16] including the time-delay, lead-lag, linear quadratic Gaussian, H_∞, H_2, and observer-based controllers.

The first obstacle to model-based control design is lack of accurate physics-based predictive models for combustion system response. Therefore, the control design typically utilizes either a model obtained from or calibrated with experimental data[2,3,9,11,14,15] or an adaptive scheme[17–20] to tune the controller parameters automatically in a way that reduces pressure oscillations. In particular, an adaptive scheme is needed as a preliminary control algorithm in the case where obtaining a data-based model is not practically feasible. This would be the case of an unstable industrial combustor operating in a regime where hardware damage is likely or when operating conditions vary (as in power transients for industrial gas turbines) and are subject to unknown disturbances (external temperature, power load changes). Both fixed-parameter and adaptive control approaches have their own limitations, which we will discuss next.

As mentioned earlier, the level of suppression achieved with fixed-parameter controller varied between various experiments. In this paper we will explain, using methods of control theory, how large transport delay (comparable with the acoustic period) and limited actuator bandwidth reduce the achievable attenuation level of pressure oscillations. In essence, in the presence of a large delay, attenuation of pressure oscillations at certain band of frequencies is accompanied by excitation of oscillations in adjacent bands.[15,21,22] Limited actuator bandwidth prevents the possibility of compensation for this problem in the control algorithm. The tradeoff between the attenuation of oscillations in certain frequency bands and excitation in adjacent bands is expressed in terms of a controller-independent lower bound[15,21,23,24] on the function that shows maximum pressure magnitude magnification over the excitation band caused by the controller. The lower bound is an increasing function of the transport delay and a decreasing function of the actuator bandwidth.[15,21]

The limitations of adaptive control algorithm performance include those of the fixed-parameter controllers with few extra limitations introduced by the adaptation. First, unless the combustor transient timescale is an order of magnitude slower than the adaptation timescale, the stability of any adaptive scheme cannot be guaranteed.[25–27] (For some ad hoc adaptive schemes, there are no stability guarantees even under assumption of the timescale separation.) Unfortunately, one cannot arbitrarily decrease the control parameter adaptation timescale to achieve the timescale separation. One factor that limits the speed of adaptation, especially in the industrial applications, is the noise present in the pressure time traces[17] that can be attributed to the response of the acoustic modes to random disturbances (such as turbulence). This noise needs to be filtered out so that the control algorithm can distinguish the pressure reaction to the control input from a response to random disturbances. The presence of the noise filters necessarily slows down the speed of adaptation as the time required to average out the effect of noise is proportional to the noise/signal ratio.[17,28] This explains why adaptive controllers demonstrated in laboratory-scale combustors (low noise) may not perform as well in industrial (high noise) settings. There are also tradeoffs between the performance, stability, and speed of adaptation of adaptive algorithms.[28]

This paper will cover a more in-depth examination of several of these factors, including actuated fuel mixing, actuation time delay, and fundamental control limitations. These factors were chosen based on an estimation of their criticality in the active control system development process. This assessment was made based on the results of diagnostic experiments, detailed physical modeling, and reduced-order dynamic modeling.

Description of the Combustor

This paper will discuss experimental results from two different experimental combustors (Figs. 1 and 2): 1) single-premixer flame tube with natural gas fuel (4 MW) and 2) three-premixer sector rig with liquid diesel fuel (12 MW).

The experiments were operated over a wide range of equivalence ratios and at inlet temperatures and pressures corresponding with real engine operating conditions (nominally 710 K and 1.5 MPa, respectively). The experiments used similar embodiments of the same engine-scale premixing nozzles. The three-nozzle sector rig used a 60-deg arc sector of the engine combustor liner with convectively cooled sidewalls. The combustor rigs are discussed in detail in Refs. 1, 2 and 29.

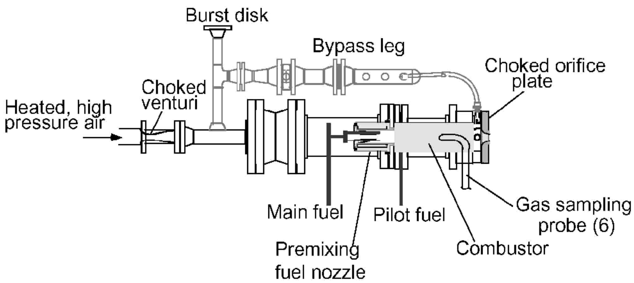


Fig. 1 Schematic of single-nozzle combustor flame tube, 15.2-cm test section diameter, for clarity, one of six sampling probes shown.

The premixing fuel injector used in these combustors has been described in detail by Stufflebeam et al.[30] The fuel nozzle is shown schematically in Fig. 3. Air was delivered into the premixing chamber through two tangentially oriented air slots that ran the entire axial length of the chamber. Natural gas fuel was injected through a row of orifices in the inlet section to each of these air slots. Fuel/air mixing was measured and optimized for low-emissions operation, as described by Stufflebeam et al.[30] The liquid-fuel version of the injector used a series of six axial spokes to atomize and inject fuel in the interior of the premixer.

The instability mode (~200 Hz) to which control systems were applied was a Helmholtz mode, $n = 0$, in which the fluctuating pressure was uniform within the combustor. Pressure fluctuations were coupled with the heat release process through their effect on the flow rate of air delivered through the premixer. The time-varying airflow rate produced a time-varying equivalence ratio at the fuel nozzle exit and, therefore, time-varying heat release rate. This conceptual model of the instability was discussed in more detail by Peracchio and Proscia,[31] and the phenomenon has been described by a number of other authors.[5,6,32] Fuel concentration measurements performed by Lee and Anderson[33] in this combustor confirmed this link between equivalence ratio fluctuations and pressure fluctuations. Figure 4 shows the spectrum of the fluctuating pressure for this instability, as observed in the single-nozzle (flame tube) version of the combustor.

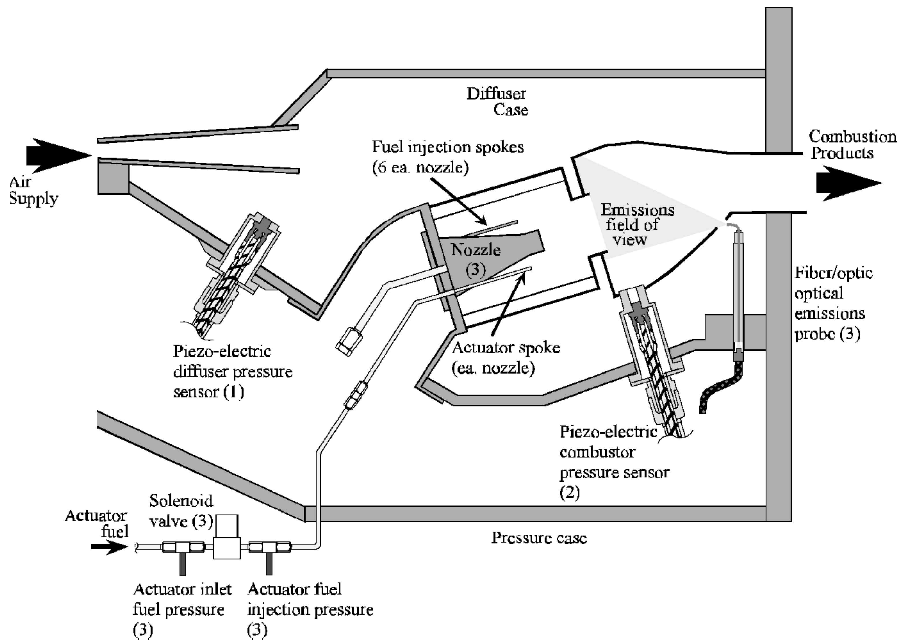
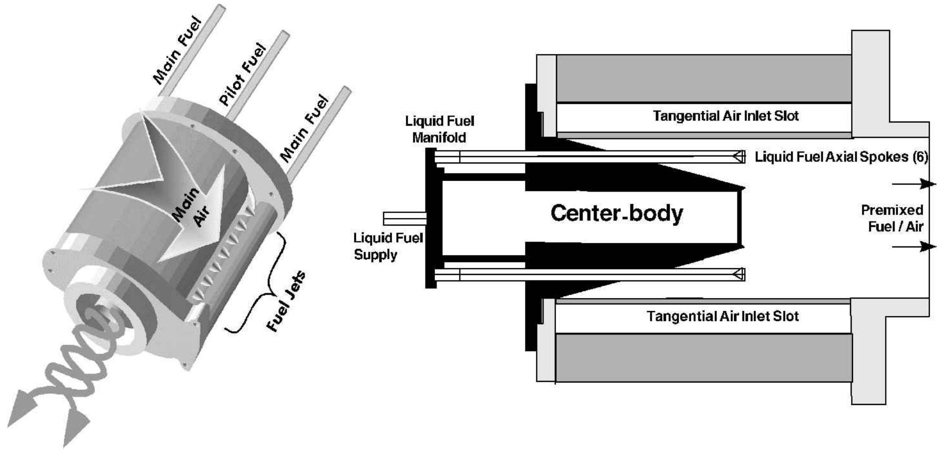


Fig. 2 Cross section of three-premixer sector combustor test facility with instrumentation and actuation system.



a) b)

Fig. 3 Schematic of premixing fuel nozzle: a) tangential air scrolls and gaseous-fuel injection scheme and b) nozzle cross section with liquid-fuel injection scheme.

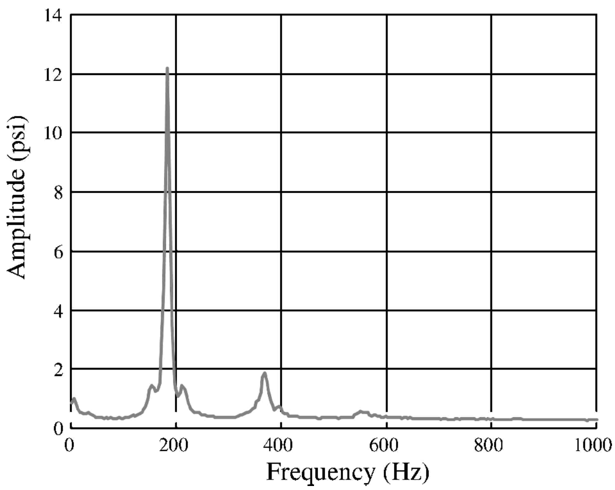


Fig. 4 Spectrum of uncontrolled combustion instability, showing high-amplitude pressure fluctuations at approximately 200 Hz.

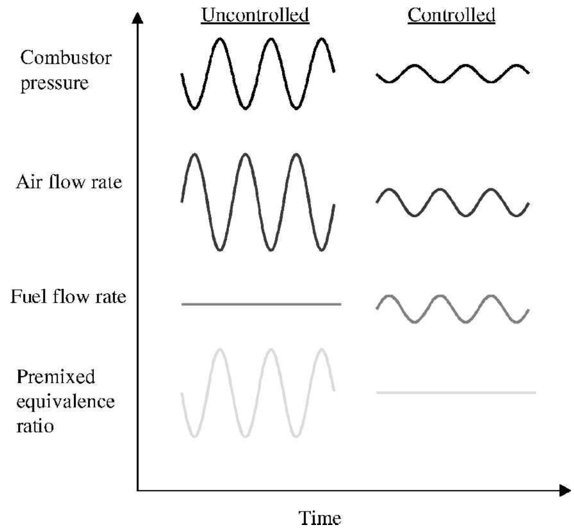


Fig. 5 Intent of the actuation technique, utilizing the reduction of equivalence ratio fluctuations by pulsed fuel injection.

The fundamental problem of control system design for low-emissions combustors is to maximize the system's authority over the relevant dynamic processes while minimizing the combustor's emissions and ensuring this performance over a range of operating conditions. The active control system consisted of three parts: a pressure sensor, a control algorithm, and an actuator. Because of the uniform spatial distribution of the fluctuating pressure within the combustor, only one combustor pressure measurement was required to describe the unsteady pressure field as input to the control system. In both experiments, on/off actuation of a portion of the fuel flow to the premixer was performed. A closed-loop control algorithm was developed to use the actuators' authority to damp combustor pressure oscillations. The control algorithm chosen consisted of a frequency-tracking observer implemented in software that identified the frequency and in-phase and quadrature components of the combustor pressure oscillations from the high-response pressure signal. The phase-shifted pressure oscillation signal was then fed back to the on/off control valve.

Actuated Fuel Mixing

Actuation technology is often identified as a critical-path item for the product deployment of active instability control systems in real engines. The majority of successful efforts in this area have used modulation of some sort of fuel flow as an actuation technique. These techniques have used existing fuel system components or have added secondary fuel injectors. The obvious actuation technology barrier is represented by the ability to modulate large fuel flows at the high frequencies at which combustion instabilities occur (normally hundreds of hertz in gas turbines). In some cases, the acoustics of the fuel system have been tuned to compensate for poor actuator performance in the frequency range of interest.[13] A second, and less obvious, barrier that relates to actuation is the physics of the actuation process. For an actuator to be effective, it must have the ability to interfere with the coupling process between the acoustic pressure field and the combustion heat release rate. An actuator that modulates fuel flow may not have significant authority (effectiveness) if the fuel is not injected in a manner that allows it to interfere with the coupling process.

The intent of the actuation scheme used in this effort is shown in Fig. 5. The controlled fuel injection system was used to reduce variations in the fuel/air ratio at the exit of the premixer by introducing a modulated fuel flow into it. The goal of the system was to keep the fuel/air ratio being delivered to the combustor uniform in both time and space. Temporal uniformity inhibits the development of combustion instabilities, and spatial uniformity inhibits the production of NO_x. Complete cancellation of fuel/air variations would likely require on-line adaptation of the amplitude of the fuel flow modulation. Only fixed-amplitude (on/off) actuation was used in this study.

The gas-fuel premixer was modified to incorporate four different actuated fuel injection configurations, as shown in Figs. 6 and 7. These injection configurations were designed specifically to provide different levels of mixing between the actuated fuel flow and the remainder of the premixed reactants. Three of the configurations used an axially oriented spoke mounted on the premixer centerbody. The length of the spoke and the number of injection sites were varied to modify mixing. The fourth configuration modulated the flow through two of the injection orifices in the main fuel injection array at the inlet to the air scroll. In all cases, the level of actuated fuel flow was held constant.

Nonreacting Injector Evaluation

A nonreacting acetone planar laser-induced fluorescence (PLIF) technique[15] was used to assess the steady-state mixing features of each injection concept. The fuel flow was not modulated during these tests. In each of the tests, 10% of the total fuel simulant injected into the premixer was passed through the control fuel injection system. In all cases, controlled fuel flow was injected inside the premixing nozzle, although in different fashions. The results of these tests are shown in Fig. 8. The concentration at the exit of the premixing nozzle is shown for each configuration. For three injectors, the nonuniformity created by the localized injection can be observed in the concentration profiles as a locally richer spot. Figure 8 also shows the spatially averaged unmixedness, as represented by the ratio of the standard deviation of the concentration distribution σ to the mean concentration value μ, for each configuration.

Three of the injection concepts (configurations 1–3) demonstrated poorer steady-state mixing than the baseline premixer. It can be seen that, for those configurations, mixing improved as the fuel simulant was injected through a larger number of sites over a larger area. Injection through the original orifice array gave the best mixing, which is consistent with the fact that this array had been optimized for good mixing.

Phase-locked PLIF measurements[29] were made for the best-mixing configuration, which used the main orifice array (configuration 4). A high-speed on/off valve in the control fuel system was driven at 200 Hz. The PLIF imaging camera was synchronized with the valve actuation, and five different time/phase delays were used over the full cycle of injection at 200 Hz. There were 1800 images acquired at each delay setting and averaged together to generate a representative, ensemble-averaged image of the concentration distribution at the exit of the premixer through a cycle of actuation. Figure 9 shows these results. They demonstrate that the spatial concentration distribution changed only slightly over the period of one cycle. Figure 10 shows how the spatially averaged concentration (fuel/air ratio) changed over one cycle. The unmixedness

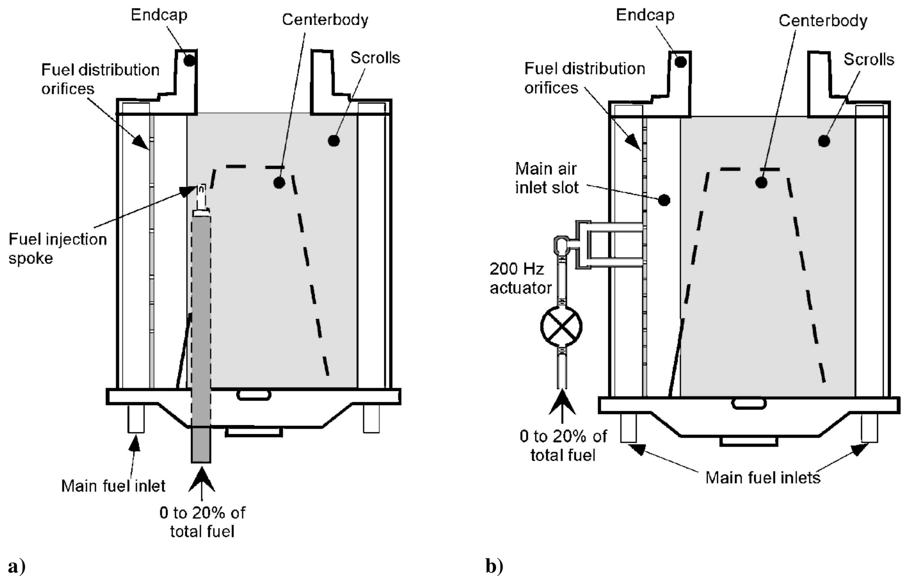


Fig. 6 Cross section schematic of premixing fuel nozzle with actuated fuel injection spoke and actuated portion of the main injection array for introduction of control fuel flow: a) configuration 1–3 and b) configuration 4.

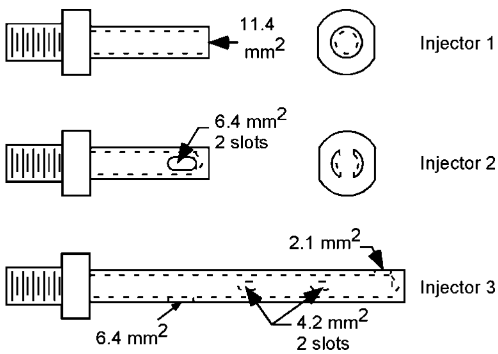


Fig. 7 Detailed view of three spoke injection configurations.

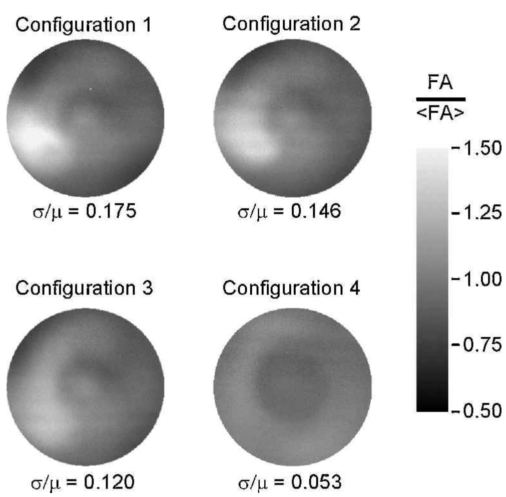


Fig. 8 Steady-state fuel/air concentration profiles at premixer exit for four fuel injection concepts; concentration values normalized by the mean value for each case.

($\sigma/\mu =$ standard deviation/mean) changed over the range of 3.6–7.9% during the cycle, indicating that reasonably good mixing was being maintained throughout. The average acetone concentration changed by $\pm 7\%$ during a cycle, compared to the command variation of $\pm 10\%$. These tests demonstrated the ability of injector configuration 4 to control the temporal character of the fuel/air ratio at the exit of the premixer without excessively degrading the spatial fuel/air mixing at any point in time.

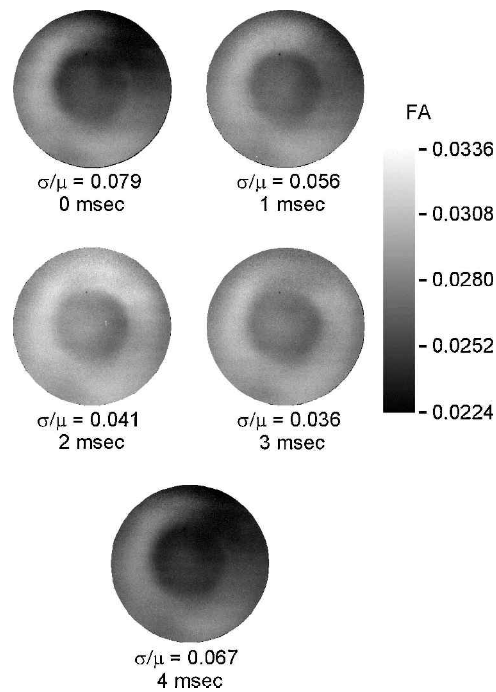


Fig. 9 Phase-averaged fuel/air concentration profiles and unmixedness at premixer exit for 200-Hz modulation of fuel injection using configuration 4.

Combusting Injector Evaluation

Each of the control fuel injection concepts was tested in the gas-fueled, single-nozzle flame tube combustor under controlled and uncontrolled conditions. During uncontrolled operation, fuel was delivered through the control fuel circuit at the same mean flow rate as was used in the controlled tests. This was necessary to isolate the effect of the unsteady aspect of the fuel injection.

Three of the poorer mixing injection concepts showed increased NO_x and CO levels (relative to baseline levels) during operation with steady, nonmodulated flow through the control fuel system. These levels did not change appreciably when control was applied (for the same mean flow through the control fuel system). In general, the level of NO_x increase that was observed correlated with the degree of unmixedness that was observed. Emissions were measured using a ganged array of six water-cooled sampling probes near the exit of the combustor.

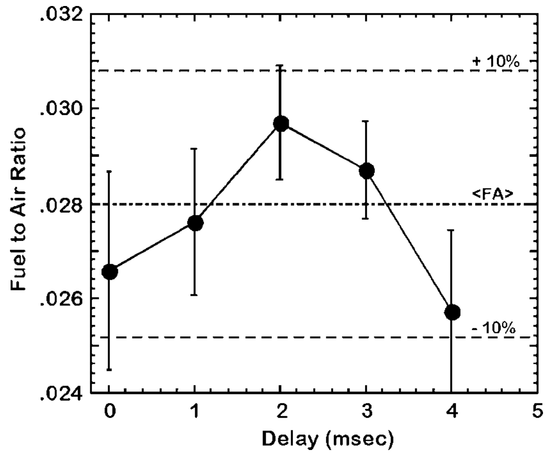


Fig. 10 Variation of spatially averaged fuel/air ratio at premixer exit over one period (5 ms) of 200-Hz fuel flow modulation for configuration 4; bars indicate spatial variance about average value at each time.

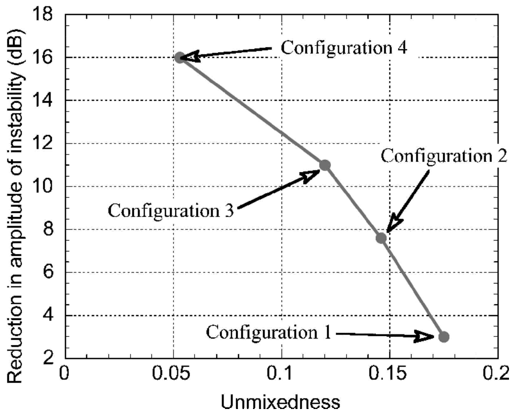


Fig. 11 Effect of steady-state fuel/air mixing on actuator authority in combustion tests for four different actuated fuel injection configurations.

For each of injection configurations 1–3, a piloting effect on the instability was also observed. When a constant fuel flow was delivered through the control fuel system, the local enrichening resulted in an attenuation of the pressure fluctuations. This behavior is similar to that observed when a diffusion-flame pilot was applied to the system. When piloted, the pressure fluctuation levels were reduced, but at the expense of higher NO_x emissions.

Configuration 4, in which the control fuel was delivered through a portion of the main fuel injection array, showed no difference in either pressure fluctuations or emissions between the baseline case and the case in which steady fuel flow was delivered through the control fuel system (as expected).

Figure 11 shows that the level of reduction in the amplitude of the dominant instability mode correlated with the mixing performance, when measured at the optimal control phase delay for each configuration. Better mixing injection schemes provided significantly better actuation authority, with configuration 4 producing a 16-dB (6.3 times) reduction. The broadband rms pressure fluctuation level in the 0–2 kHz band was reduced by a factor of 2.4 with this control.

One of the more interesting observations was that both NO_x and CO emissions were improved under controlled conditions for configuration 4. NO_x emissions were reduced by 27%, and CO emissions were reduced by 54%. This trend is consistent with observations made by Cohen et al.[2] It is most likely that the control system, by reducing equivalence ratio fluctuations, also reduced temporal hot spots and cold spots, thus, decreasing the production of NO_x and CO, respectively. These effects are substantial due to the highly nonlinear relationship between the pollutant emission production rates and flame temperature at these low equivalence ratios.

Actuation Time Delay

Another consideration that affects the authority of actuation systems is the time delay between when the actuator moves and when the effect of that actuation is observed at the flame front. This was investigated using both the liquid-fueled three-nozzle sector combustor[1] and the single-nozzle flame tube combustor. Semi-empirical dynamic models of the system were developed that explain the origin of these effects.

Experimental Observations

Speculating that the effectiveness of the controller could be increased with the additional actuator authority produced by actuating more nozzles, the control system was tested using multiple simultaneously actuated fuel nozzles. Figure 12 shows power-spectral density (PSD) plots for phase-optimized (minimum-pressure oscillations) single, dual, and triple closed-loop controlled fuel nozzles. Combustor pressure oscillations were reduced by going from single to dual nozzle actuation, but no further reduction was obtained by actuating all three fuel nozzles, in spite of the open-loop forcing results. The best control was achieved with dual nozzle actuation, yielding a 6.5 dB (2.1 times or 53%) reduction in the bulk mode pressures and a 25% reduction in broadband rms pressure. These reductions in combustor pressure oscillations via active control were accompanied with no penalties to emissions compared to uncontrolled operation. The magnitude of the reduction was limited by the splitting of the spectral peak into two smaller peaks. This splitting behavior was evident for both two- and three-nozzle actuation, but the amplitude of the secondary peaks was larger for the three-nozzle case.

In later experiments with the gas-fueled, single-nozzle flame tube combustor, the length of tubing between the actuation fuel valve and the fuel injection location was changed from 2.8 (1.1) to 15.2 cm (6 in.) and 45.7 cm (18 in.). The effect of these variations on the control system performance is shown in Fig. 13. The negative effects of peak splitting increased as the fuel line was made longer. Although no direct measurements of time delay were made for the different configurations, it is reasonable to assume that increasing the fuel line length increased the time delay of the actuation system. This phenomenon has been experimentally observed by other investigators as well.[7]

Dynamic Interpretation of Peak Splitting

The explanation for this phenomenon traces back to the fundamental nature of the pressure oscillations observed in the combustor. Pressure oscillations in a combustor dominated by a narrow frequency band can be interpreted using a limit-cycling model or a stable, noise-driven model. Most references attribute pressure oscillations in combustor to self-excitation of coupled acoustics and heat

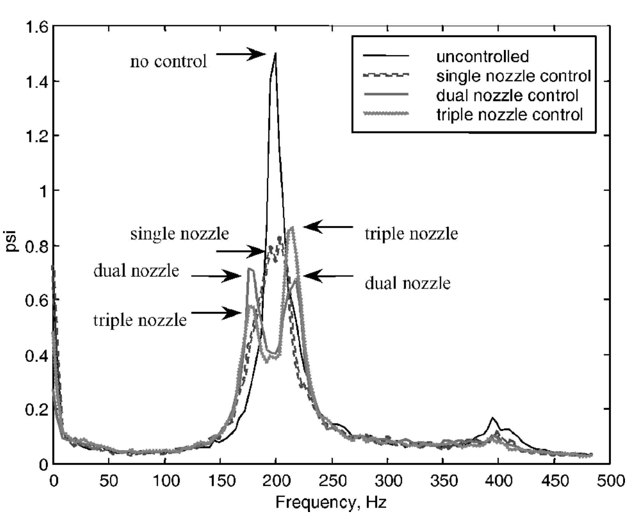


Fig. 12 Multiple-nozzle, closed-loop actuation led to relatively small incremental reductions in pressure fluctuation levels due to peak-splitting phenomenon.

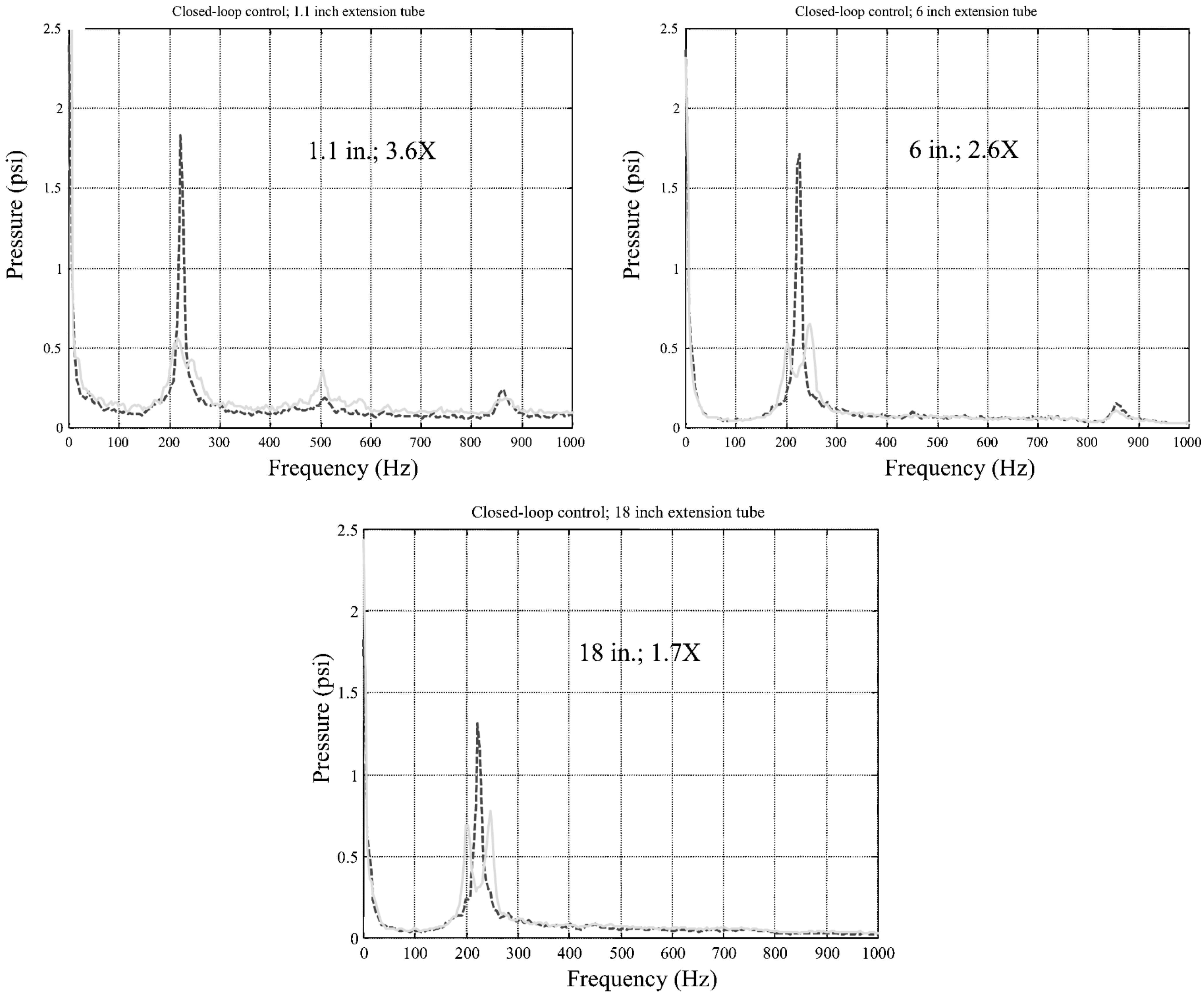


Fig. 13 Combustor pressure spectra showing the reductions in peak amplitude of optimum-phase control for three different lengths of fuel manifold with results indicating reduction in control system authority due to appearance of dual peaks at longer tubing lengths: ——, controlled cases and – – –, uncontrolled cases.

release system resulting in a limit-cycling behavior. However, it will be shown that the uncontrolled sector combustor behavior and the splitting of the bulk mode peak observed during controlled operation can be better explained using a model of the combustor as a lightly damped, linearly stable system driven by noise attributed to turbulence. Note that there are important differences between behavior observed in small laboratory combustion control experiments and full-scale industrial combustors. Laboratory combustors typically have no liner and, thus, have lower damping than industrial combustors. At the same time, laboratory combustors may have lower turbulence levels than larger, more complex devices. With low damping and low noise levels, it is likely that significant pressure oscillations will only occur due to self-excited limit-cycle oscillations.[14] Industrial combustors can exhibit noticeable pressure oscillations in a stable, noise-driven regime, and hence, a self-excited model is, in many cases, not necessary. In this sense the term combustion instability is less appropriate for this analysis because it will use a stable model of the sector rig and add a driving broadband stochastic disturbance to account for the observed pressure oscillations. Of course, in some cases, a self-excited model of pressure oscillations in industrial combustors will be more appropriate than a stable-driven model.[2,3] Note, however, that large combustor pressure oscillations are possible in both scenarios.

Experimentally determined frequency responses of the combustor pressure to the valve actuation voltage (Fig. 14) closely resembled those typical of linear systems with time delay. A second-order model with delay was used to fit these dynamics with good agreement in the frequency range of 150–400 Hz. In principle, these empirical fits are necessary but not sufficient to conclude that the combustor dynamics were in fact linearly stable (as opposed to perhaps limit-cycling behavior) because a limit-cycling system may produce a frequency response resembling that of a stable-driven system. However, there are several additional arguments that a driven stable system is indeed a good model of the observed behavior. First, the uncontrolled pressure PSD can be closely matched using a noise-driven stable model. Second, Fig. 15 shows that the probability distribution of pressure from experiments is very well approximated by Gaussian distribution, which is a typical distribution of an output of a linear system driven by Gaussian input. A noise-driven limit-cycling system would show a double-hump distribution[22,34,35] of pressure. Last, it will be shown that the stable, driven model reproduces the peak-splitting effect of the controller in simulations with encouraging fidelity to data.

The transfer function of combustor pressure to valve command voltage was measured via open-loop swept-sine tests actuating one of the three nozzles. The first step in fitting a measured transfer function was to identify the time delay from the slope of the phase in the frequency range of 220–260 Hz. Next, the phase lag due to delay was subtracted from the experimental phase lag yielding a nearly classical second-order response with the phase dropping 180 deg through the magnitude response peak. A stable second-order transfer function with two poles and one zero was fitted numerically.

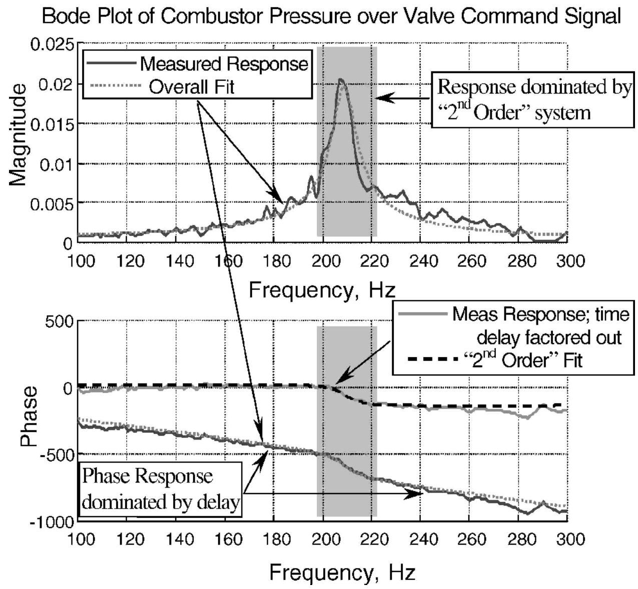


Fig. 14 Bode plot of combustor pressure over valve command signal with no control.

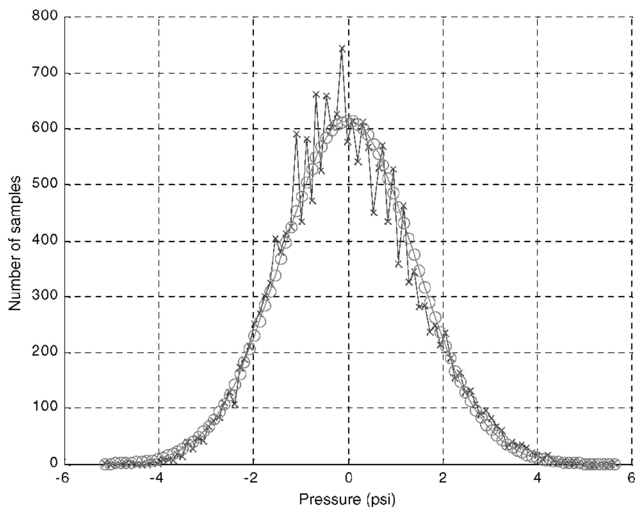


Fig. 15 Distribution of 20000 samples: ×, uncontrolled unsteady combustor pressure and ○, fit with a Gaussian distribution.

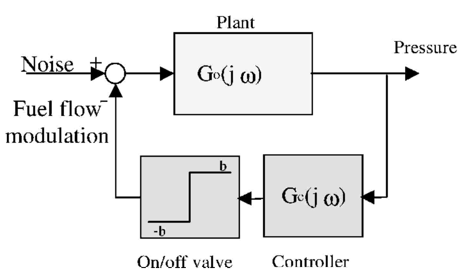


Fig. 16 Schematic of closed-loop combustor simulation block diagram; noise represented as random fluctuations of fuel/air ratio.

A schematic of the closed-loop simulation block diagram is shown in Fig. 16. The plant $G_0(j\omega)$ is the empirical second-order system with delay representing the combustor dynamics. With the controller off, the standard deviation of the white Gaussian noise was adjusted in the simulation to match the PSD of the pressure data from experiment. A likely physical source of the driving disturbance was the turbulent flow fluctuations driving the acoustic mode directly, or through the heat release process.

The effect of multiple-nozzle actuation was simulated by linearly scaling the controller output by the number of nozzles, which was

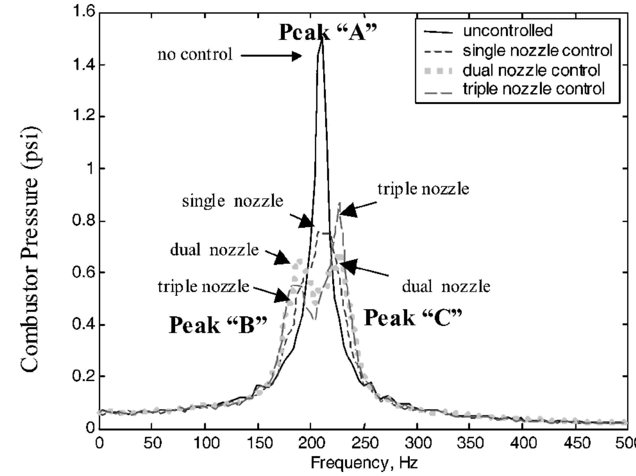


Fig. 17 Second-order model of combustor with delay reproduces peak-splitting phenomenon (Fig. 12) in closed-loop simulation.

consistent with experimental open-loop forcing results. Figure 17 shows that the simulation exhibited a peak-splitting phenomenon similar to those observed in the experiments. The amplitude at the dominant oscillation frequency was attenuated, whereas secondary peaks were amplified by actuating more nozzles and, therefore, more authority. The slight asymmetry of peaks after the third valve was turned on in the experiment can be attributed to a different phase lag of the third actuation system relative to the first two. In fact, this asymmetry was reproduced in the model by assigning a larger delay to valve three than to valves one and two in the model.

Even though a linear, stable system driven by a white Gaussian disturbance was a good model of the sector combustor during the experiments, using on/off valves for control made the closed-loop system strongly nonlinear. Thus, an analysis of peak splitting using linear control theory tools[14] may not seem immediately relevant. However, it has been argued by Banaszuk et al.[21,22] that a quasi-linear analysis using random input describing functions is appropriate to study the nonlinear dynamics of the combustion model with on/off valves in the presence of large-amplitude noise, as is the case with this closed-loop model. In this technique, the signals in the model were approximated as sums of constant, sinusoidal, and random components with Gaussian distribution. The static nonlinear elements were replaced with equivalent gains called random input describing functions.[36] The values of constant components, amplitudes, and frequencies of sinusoidal components and the standard deviations of Gaussian components in the system can be found by solving a system of nonlinear equations.

It can be shown that the system of Fig. 16 with the identified standard deviation of Gaussian input has low effective gain of the on/off valve for the sinusoidal signal so that a limit-cycle oscillation cannot be sustained. Therefore, only the balance of Gaussian processes in the loop has to be carried out to predict approximately the PSD of the combustor pressure under closed-loop control. The random input describing function for the on/off valve with on level denoted by b is

$$N(\sigma) = \sqrt{2/\pi}(b/\sigma)$$

where σ is the standard deviation of the Gaussian process at the input of the valve. One can shown that, given the noise input PSD $\Phi_{ii}(j\omega)$, σ can be found from the Gaussian process balance equation as

$$\sigma = \sqrt{\frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{G_0(j\omega)G_c(j\omega)}{1+\sqrt{2/\pi}(b/\sigma)G_0(j\omega)G_c(j\omega)}\right|^2 \Phi_{ii}(j\omega)\,\mathrm{d}\omega}$$

which can be solved numerically. Once the value of σ is known, the pressure PSD $\Phi_{pp}(j\omega)$ can be obtained from

$$\Phi_{pp}(j\omega) = \frac{G_0(j\omega)}{1+N(\sigma)G_0(j\omega)G_c(j\omega)}\Phi_{ii}(j\omega)$$

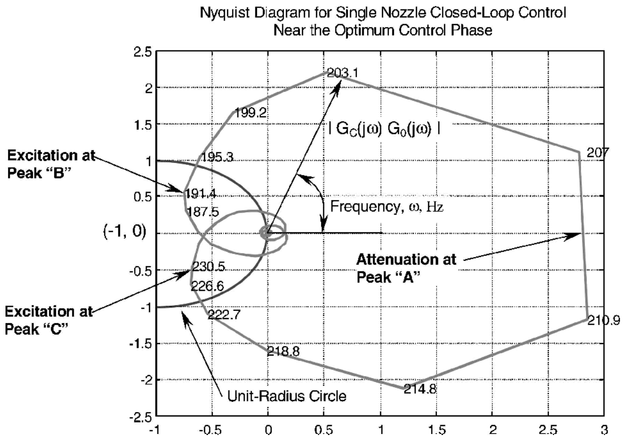


Fig. 18 Nyquist diagram for single-nozzle closed-loop control near the optimum control phase, showing that the controller excited secondary peaks B and C and attenuated the primary peak A.

The closed-loop transfer from noise to pressure is $G_0(j\omega)/[1 + G_0(j\omega)G_C(j\omega)N(\sigma)]$, indicating that for $|1 + G_0(j\omega)G_C(j\omega) N(\sigma)| < 1$ the pressure oscillations are amplified by the controller and for $|1 + G_0(j\omega)G_C(j\omega)N(\sigma)| > 1$ the pressure oscillations are attenuated by the controller.

Figure 18 shows the Nyquist plot of $G_0(j\omega)G_C(j\omega)N(\sigma)$ in the complex plane for the value of b corresponding to three valves. [For Fig. 18, $G_0(j\omega)G_C(j\omega)$ and σ were obtained from model simulation rather than calculation, the main reason being that the controller $G_C(j\omega)$ used in the experiments and in the simulation was a nonlinear phase-shifting controller based on a frequency-tracking extended Kalman filter, which does not have a simple linear transfer function, even though for a fixed central frequency of oscillations it can be closely approximated by a linear transfer function.]

By analyzing the Nyquist diagram shown in Fig. 18, we conclude that the controller amplifies certain frequency bands while attenuating the pressure oscillations at the frequency band centered at about 208 Hz (peak A in Fig. 17). The two nearly symmetric branches of the Nyquist plot that cross into the unit-radius circle, centered at $(-1, 0)$, for frequencies greater than 225 Hz and less than 195 Hz, were the root cause of the secondary peaks B and C in Fig. 17. They arise because of presence of large delay in the combustor transfer function causes significant rolloff of the phase of the open-loop transfer function $G_0(j\omega)G_C(j\omega)N(\sigma)$ Although adding more nozzles and, therefore, more actuator authority increased the control gain in the attenuation band, it also increased the control gain in the excitation band as well, imposing a limit on the phase-shifting controller's effectiveness. Note however, that increasing the number of actuated nozzles does not correspond to proportional gain increase because the standard deviation σ of the Gaussian process at the input of the valve is a function of b.

Fundamental Limitations of Achievable Performance

In the preceding section, we showed that increasing delay between the fuel control command and its effect on the combustion process reduces attenuation of the pressure oscillations in a combustor. A natural question arises whether the increase of the delay could be compensated by a choice of the control algorithm. Banaszuk et al.[21,22] showed (using the approach of Freudenberg and Looze[23]) that one cannot arbitrarily decrease the level of pressure oscillations using linear controllers. In this section we review these results. The derivation of fundamental limitation is provided for the case of a linear combustor response and linear controller transfer function. Extension to the nonlinear actuator case is discussed at the end of this section.

Recall that the combustor pressure PSD $\Phi_{pp}(j\omega)$ can be obtained from

$$\Phi_{pp}(j\omega) = |G_0(j\omega)S(j\omega)|^2\Phi_{ii}(j\omega)$$

where

$$S(j\omega) := 1/[1 + G_0(j\omega)G_c(j\omega)]$$

is the sensitivity function. Note that the square of the sentitivity function is the factor by which the PSD of pressure in the combustor is reduced (or amplified) at any given frequency. The objective for active control of combustion is to shape the sensitivity function so that it is small at and near the resonant frequency ω_r of the combustor. This requirement can be stated as

$$|S(j\omega)| < \varepsilon \qquad \text{for} \qquad \omega \in \Delta\omega_1$$

where $\Delta\omega_1$ is the so-called performance bandwidth, that is, the interval containing the resonant frequency ω_r over which reduction of pressure oscillations by the factor of ε relative to uncontrolled level is enforced. The fundamental limitations[23,24] yield controller-independent lower bounds on the maximum of the sensitivity function. Assume that the combustor response transfer function $G_0(j\omega)$ has at most one unstable complex conjugate pole pair with the real part denoted by σ_r. If the combustor model is stable, we define $\sigma_r = 0$. An example of fundamental limitations is the Bode integral formula for the sensitivity function:

$$\int_0^\infty \ln|S(j\omega)|\,\mathrm{d}\omega = 2\pi\sigma_r$$

The preceding equation shows that negative area under the logarithm of the absolute value of the sensitivity function (corresponding to attenuation of pressure oscillations relative to uncontrolled combustor) in one frequency band must be accompanied by positive area (amplification of pressure oscillations) in some other band. If the control bandwidth is infinite, the positive area may be distributed over a wide frequency range so amplification at any given frequency may be designed to be arbitrarily small. However, if the control bandwidth is finite due to factors such as actuator bandwidth [so that $G_0(j\omega)G_C(j\omega)$ is close to zero beyond certain low and high frequencies], the positive area would have to be accommodated in a smaller band (where loop gain is high), and this would necessarily result in peaking of the sensitivity function. If the peaking occurs in the region where the combustor response transfer function has a nonvanishing gain, the peaking in the sensitivity function will result in a peak splitting in the closed-loop response. Figures 19 and 20 illustrate this phenomenon.

Assume that the combustor transfer function $G_0(j\omega)$ is of relative degree of at least two, that is, it has at least two more poles than zeros. This assumption is typically satisfied if actuator and sensor dynamics are included in combustor response transfer function. To model the effect due to finite control bandwidth, we require the open-loop gain to satisfy the inequality

$$|G_0(j\omega)G_c(j\omega)| \le \delta(\omega_c/\omega)^{1+k} \qquad \text{for} \qquad \omega > \omega_c$$

Here, it is assumed that $\delta < \frac{1}{2}$ and $k > 0$ (relative degree of at least two). We impose a similar constraint on the loop gain,

$$|G_0(j\omega)G_c(j\omega)| \le \delta(\omega/\omega_b)^{1+k} \qquad \text{for} \qquad \omega < \omega_b$$

Let us define the control bandwidth $\Delta\omega_2 := \omega_c - \omega_b$. Figure 21 shows the finite bandwidth performance specification with the performance and control bandwidth. The restrictions of the loop gain at

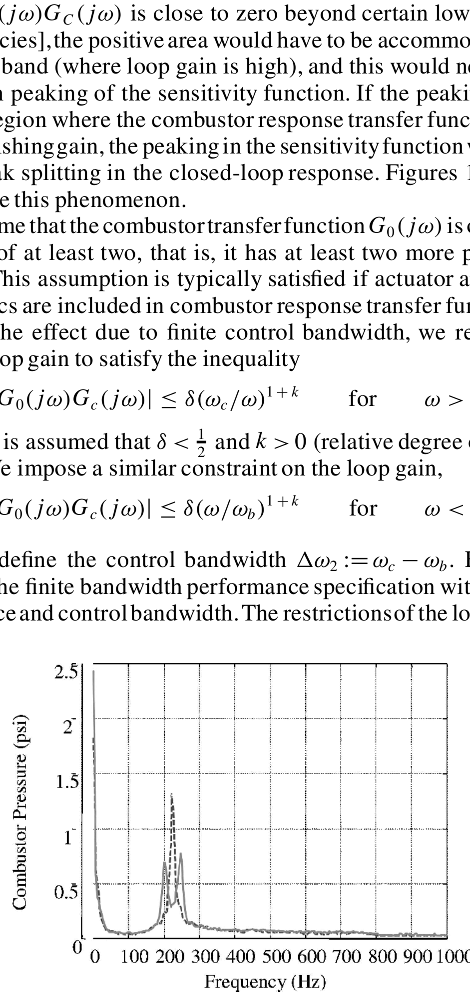


Fig. 19 Peak-splitting phenomenon, showing sidebands on either side of uncontrolled peak during controlled operation: ——, controlled case and – – –, uncontrolled case.

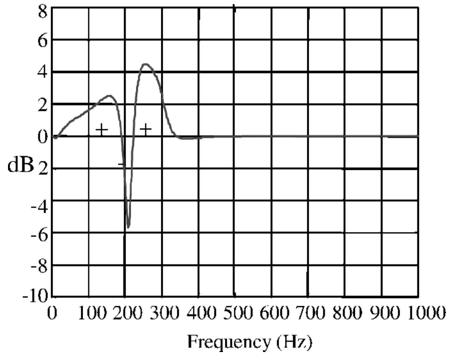


Fig. 20 Typical sensitivity function, showing the sensitivity tradeoffs due to finite controller performance bandwidth.

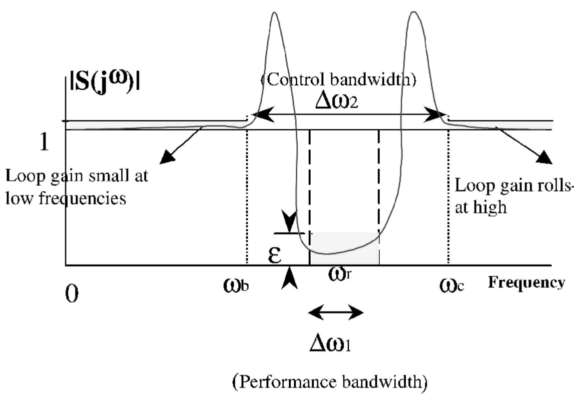


Fig. 21 Performance and control bandwidth.

high and low frequencies impose additional constraints on the sensitivity function. Now, in addition to the performance specification $|S(j\omega)| < \varepsilon$ for $\omega \in \Delta\omega_1$, a control bandwidth specification must also be met.

We now compute the performance limitations as peaking in the sensitivity function magnitude. Let $\|S\|_\infty := \mathrm{Sup}_\omega |S(j\omega)|$ denote the so-called H_∞ norm of the sensitivity function. Note that $\|S\|_\infty$ is the supremum over all frequencies of the amplification of the pressure oscillations by the control system relative to the uncontrolled response, that is, a measure of control-induced peaking. For the finite control bandwidth case, the area formula together with the constraints shown in Fig. 21 and high-frequency rolloff characteristics can be manipulated (as Ref. 23) to show that

$$\log \|S\|_\infty \geq \frac{1}{\Delta\omega_2 - \Delta\omega_1}\left\{2\pi\sigma_r + \Delta\omega_1 \log\frac{1}{\varepsilon} - \omega_b \log\frac{1}{1-\delta} - \frac{3\delta\omega_c}{2k}\left[1 - \frac{1}{(1+\pi/\tau\omega_c)}\right]\right\}$$

The preceding formula shows the following factors that bound from below the supremum of the sensitivity function.

1) The desired performance is represented by the product $\Delta\omega_1 \log(1/\varepsilon)$.

2) The limitation on the actuator bandwidth relative to required performance bandwidth is represented by the amplifying term $1/(\Delta\omega_2 - \Delta\omega_1)$.

3) The real part of the unstable combustor pole is represented by the $2\pi\sigma_r$. The larger the growth rate of the pole, the larger is the peak of the sensitivity function.

4) The combustion response delay is τ. One can verify that the lower bound on the sensitivity peak is an increasing function of the delay (assuming other parameters are fixed).

With use of the inequality on the sensitivity peaking, Figs. 22–24 show lower bounds on sensitivity function norms. Figures 22–24 show that, as the ratio of the control bandwidth to the performance bandwidth decreases, the sensitivity peaking becomes more and more severe. Furthermore the peaking is accentuated by increase in the delay τ, increase in the performance requirements (lower ε), or increase in the real part of the unstable pole of the open-loop plant σ_r.

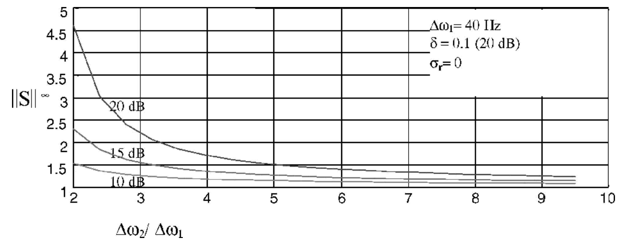


Fig. 22 Lower bounds on sensitivity function norm as function of control bandwidth for three values of ε.

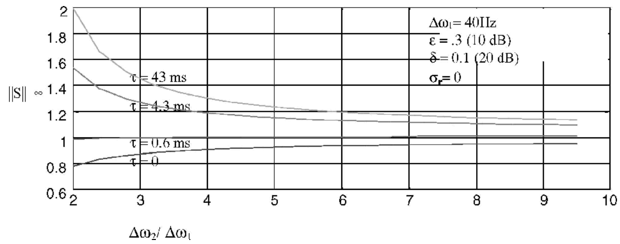


Fig. 23 Lower bounds on sensitivity function norm as function of control bandwidth for four values of τ.

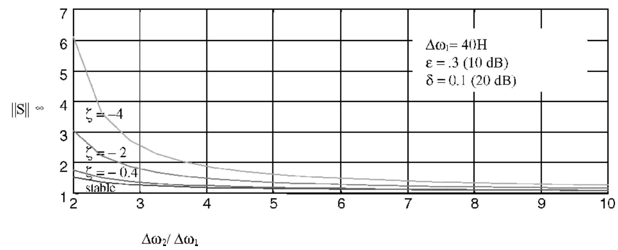


Fig. 24 Lower bounds on sensitivity function norm as function of control bandwidth for four values of combustor pole damping ratio.

Extension of fundamental limitations to the case when either plant or controller has nonlinear characteristics is possible using the concept of random input describing functions introduced in the preceding section. For example, in the case of sector combustor controlled with on/off valves, the fundamental limitations in terms of lower bounds on the logarithm of the sensitivity function as presented in this section applies with $G_C(j\omega)$ replaced with $G_C(j\omega)N(\sigma)$, where $N(\sigma)$ is the random input describing function of the on/off valve and σ is the standard deviation of the Gaussian component of the valve command. Fundamental limitation analysis also extends to the cases when more general nonlinearities are present in the model of combustor or controller, Gaussian noise sources are present, and the combustor feedback loop operates at a limit cycle. In each particular case, one has to prove existence of the Gaussian and periodic signals in the feedback loop that provide balance and examine stability of solution. More details are presented in Ref. 21.

Summary

This paper has discussed some of the factors that may limit the performance of active control systems for the attenuation of combustion instabilities. A broad range of factors was identified and discussed. This list is by no means complete. Many effects can be system dependent; they are critical to one system, but irrelevant for another. This may vary depending on the nature of the control system architecture and on the combustion dynamics that are being controlled. Three of the critical factors limiting the control of the lean, premixed combustor design considered in this paper have been examined in more detail.

The ability of the fuel actuator to affect the root-cause physics behind pressure/heat release coupling was found to be tied strongly to the mixing of the actuated fuel flow with the remainder of the premixed reactants. In effect, the actuated fuel flow must act to achieve a high degree of premixedness, both in time and space.

Another limiting factor relates to actuation time delay, as represented by the time between movement of the fuel valve and realization of that fuel flow modulation in the unsteady heat release or combustor pressure. Large values of time delay were found to shrink the frequency band over which the control system could attenuate pressure oscillations and to provide a mechanism through which pressure oscillations outside of this bandwidth could be amplified. This led to the peak-splitting phenomenon that limited the degree to which pressure oscillations could be suppressed.

Attempts to deal with these issues led to a sensitivity function analysis of the fundamental limits of combustor pressure oscillation control. It was shown that that one cannot arbitrarily decrease the level of pressure oscillations using linear controllers. This limit is strongly influenced by system time delays, control bandwidth, and performance bandwidth.

Although these represent current limitations, they are certainly not the only factors that will affect performance of instability control systems. Other systems (not of this type) will face different issues, and different factors may control their performance. Active combustion instability control has been well demonstrated as a technology with significant potential. For the technology to mature to the point of being practically applicable, future efforts must focus on these limiting factors, quantify them, and devise methods for dealing with them.

Acknowledgments

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